Unit 15
Day 2
The Remainder Theorem

12) $\left(\frac{1}{3}x^3-\frac{2}{9}x^2+\frac{1}{27}x+1\right) \div \left(x-\frac{1}{3}\right)$

$$\begin{array}{c|cccc} \frac{1}{3} & \frac{1}{3} & -\frac{2}{9} & \frac{1}{27} & 1 \\ & & \frac{1}{9} & -\frac{1}{27} & 0 \\ \hline & \frac{1}{3} & -\frac{1}{9} & 0 & 1 \end{array}$$

$\frac{1}{3}x^2-\frac{1}{9}x+\left(\frac{1}{x-\frac{1}{3}}\right)\frac{3}{3} = \frac{1}{3}x^2-\frac{1}{9}x+\frac{3}{3x-1}$

18) $f(x) = 2x^4+x^3-15x^2+3x \;; -3$

$$\begin{array}{c|ccccc} -3 & 2 & 1 & -15 & 3 & 0 \\ & & -6 & 15 & 0 & -9 \\ \hline & 2 & -5 & 0 & 3 & -9 \end{array}$$

$f(x)=(2x^3-5x^2+3)(x+3)-9$

Remainder Theorem: If the polynomial P(x) is divided by x-k the remainder is P(k)

1) P(x)=x³+8x²-5x-84   k=-5

$P(-5) = (-5)^3+8(-5)^2-5(-5)-84$
$= -125+200+25-84$
$= 100-84$
$P(-5) = 16$ Remainder $\frac{P(x)}{x+5}$

$$\begin{array}{c|cccc} -5 & 1 & 8 & -5 & -84 \\ & & -5 & -15 & 100 \\ \hline & 1 & 3 & -20 & 16 \end{array}$$

$P(-5)=16$

2) $f(x)=2x^3+3x^2-8x-12$ $k=-2$ What is $f(-2)$?

$f(-2)=2(-2)^3+3(-2)^2-8(-2)-12$

$=2(-8)+3(4)+16-12$

$=-16+12+16-12$

$=0$

$f(-2)=0$

$$\begin{array}{r|rrrr} -2 & 2 & 3 & -8 & -12 \\ & & -4 & 2 & 12 \\ \hline & 2 & -1 & -6 & 0 \end{array}$$

$f(-2)=0$

A ZERO of a polynomial f(x) is a number k, such that f(k)=0. Zeros are sometimes called roots or solutions of the function. The zeros (or roots) are the x-intercepts of the graph of the function.

Tell whether the given number is a zero of the given polynomial.

1) 1; $f(x)=x^3-4x^2+9x-6$

$$\begin{array}{r|rrrr} 1 & 1 & -4 & 9 & -6 \\ & & 1 & -3 & 6 \\ \hline & 1 & -3 & 6 & 0 \end{array}$$

Yes

Tell whether the given number is a zero of the given polynomial.

2) -4; $f(x)=x^4+x^2-3x+1$

Tell whether the given number is a zero of the given polynomial.

3) $1+2i;\quad f(x)=x^4-2x^3+4x^2+2x-5$

$1+2i$	1	-2	4	2	-5
		$1+2i$	-5	$-1-2i$	$1+4$
	1	$-1+2i$	-1	$1-2i$	0

Yes

$f(1+2i)=0$

When is the remainder theorem useful?

Is $P(x)=-2x^{19}+8x^{17}-6x^{10}+x^8+12$ divisible by $x-2$?

$P(2)=-2(2)^{19}+8(2)^{17}-6(2)^{10}+2^8+12$

$=\cancel{-2^{20}+2^{20}}-3(2)^{11}+2^8+12$

$=-5876$ No

Is $P(x)=x^{26}-6x^{18}+3$ divisible by $x-i$

$P(i)=i^{26}-6i^{18}+3$

$=-1+6+3$

$=8$ No

Extra problems:

1) Is $P(x)=x^{99}-2x^{52}+x^2$ divisible by $x+1$?

2) Is $P(x)=x^{101}+3x^{20}+x^3$ divisible by $x-i$?

3) Find the value of k so that $(x^2+4x+8)\div(x-k)$ has a remainder of 4.

HW pg 290-291 1-4 all, 20-30 even & Extra problems